

Semiconductor Process (EB68684)

Appendix 2: Derivation of Sheet Resistance from Four Point Probe Method

Prof. Sungsik Lee

Department of Electronics, Pusan National University

■ Derivation for $t \ll s$:

Starting with the Ohm's law,

$$J = \sigma E = \frac{E}{\rho} \quad (1)$$

Here, J is defined at a certain distance from the source current (I) with a ground at infinity, as follows,

$$J = \frac{I}{2\pi r t} \quad (2)$$

Here, we assumed that the cylindrical current flowing since $t \ll s$.

And the E-field intensity can be independently connected with potential (V) through the gradient,

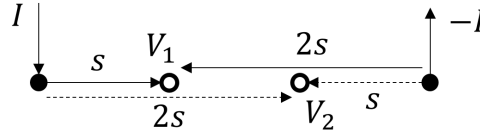
$$E = -\frac{dV}{dr} \quad (3)$$

From Equations (1) to (3), we get the equation as follows,

$$dV = -\left(\frac{I\rho}{2\pi t}\right)\left(\frac{1}{r}\right) dr \quad (4)$$

By integrating Equation (4), we get

$$V = -\left(\frac{I\rho}{2\pi t}\right)(\ln r + C) \quad (5)$$



Now, looking at the figure above, we assuming that measuring point 1 is at $r=s$ from the source current (I), Eq.(5) becomes,

$$V_{source} = -\left(\frac{I\rho}{2\pi t}\right)(\ln s + C) \quad (6)$$

Since we have the sink current ($-I$) and the measuring point 1 is at $r=2s$ from it, Eq.(5) becomes,

$$V_{sink} = +\left(\frac{I\rho}{2\pi t}\right)(\ln 2s + C) \quad (7)$$

Thus, the total potential at this measuring point 1 is given by the sum of Eqs.(6) and (7) by the superposition principle,

$$V_1 = V_{source} + V_{sink} = \left(\frac{I\rho}{2\pi t}\right)(\ln 2s - \ln s) \quad (8)$$

Similarly, the total potential at another measuring point 2 is given by superposition principle,

$$V_2 = V_{source} + V_{sink} = \left(\frac{I\rho}{2\pi t}\right)(\ln s - \ln 2s) \quad (9)$$

Finally, the voltage drop across the point 1 and point 2 is given as,

$$V = V_1 - V_2 = \left(\frac{I\rho}{2\pi t}\right)(2 \ln 2s - 2 \ln s) = \left(\frac{I\rho}{\pi t}\right) \ln 2 \quad (10)$$

So, the sheet resistance is derived from Eq.(10),

$$R_{sheet} = \frac{\rho}{t} = \frac{V}{I} \frac{\pi}{\ln 2} \quad (11)$$